πN *scattering in relativistic BChPT revisited*

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Part I

Introduction

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  - HBChPT **[Jenkins and Manohar, PLB 255 (1991) 558]** : Lorentz invariance is lost, does not converge in the subthreshold region **[Bernard, Kaiser, Meissner, Int.J.Mod.Phys.E4:193-346,1995]**, **[T. Becher and H. Leutwyler, JHEP 0106 (2001) 01]** $\Rightarrow$ We cannot check Chiral symmetry predictions for QCD.
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Part II

Formalism

Formalism

We consider the process $\pi^a(q)N(p,\sigma;\alpha) \to \pi^{a'}(q')N(p',\sigma';\alpha')$ descomposing the amplitudes in the usual Lorentz and isospin-invariant form:

$$\begin{aligned} T_{aa'} &= \delta_{aa'}T^+ + \frac{1}{2}\left[\tau_a,\tau_{a'}\right]T^- \\ T^\pm &= \bar{u}\left(p',\sigma'\right)\left[A^\pm + \frac{1}{2}(\not{q}+\not{q}')B^\pm\right]u\left(p,\sigma\right) \end{aligned}$$

We assume isospin symmetry and consider the states with definite isospin $I = 3/2$ and $I = 1/2$, and definite total angular momentum J and orbital angular momentum ℓ:

$$T_{IJ\ell}(s) = \frac{1}{\sqrt{4\pi(2\ell+1)}(0\sigma\sigma|\ell\frac{1}{2}J)}\sum_{m,\sigma'}\int d\hat{\vec{p}}'(m\sigma'\sigma|\ell\frac{1}{2}L)$$
$$\times\, Y_\ell^m(\hat{\vec{p}}')^*\langle\pi(-\vec{p}';a')N(\vec{p}',\sigma';\alpha')|T|\pi(-\vec{p};a)N(\vec{p},\sigma;\alpha)\rangle_I$$

For the calculation of the πN amplitude up to $\mathcal{O}(p^3)$, we use the chiral Lagrangian:

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)}$$

Where the superscript indicates de chiral order and $\mathcal{L}_{\pi\pi}^{(n)}$ and $\mathcal{L}_{\pi N}^{(n)}$ corresponds to a pure mesonic Lagrangian and a Lagrangian with baryons, respectively, of chiral order n.

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That Lagrangians have the following form:

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$
$$\mathcal{L}_{\pi\pi}^{(4)} = \frac{1}{16}\ell_4\left(2\langle u_\mu u^\mu\rangle\langle\chi_+\rangle + \langle\chi_+\rangle^2\right) + \dots$$

Where the ellipsis refers to terms not needed in the calculations given here and $\langle \dots \rangle$ refers to the trace over the isospin matrices.. F is the pion weak decay constant in the chiral limit and

$$u^2 = U \quad , \quad u_\mu = i u^\dagger \partial_\mu U \; u^\dagger \quad , \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$U(x) = \sqrt{1 - \frac{\vec{\pi}(x)^2}{F^2}} + i\frac{\vec{\pi}(x)\cdot\vec{\tau}}{F} \text{ (Non-lineal sigma parametrization)}$$

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Formalism

And the πN Lagrangians:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}(i \not{D} - \mathring{m})\psi + \frac{g}{2}\bar{\psi} \not{u} \gamma_5 \psi \ ,$$

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \langle \chi_+ \rangle \bar{\psi}\psi - \frac{c_2}{4m^2}\langle u_\mu u_\nu \rangle (\bar{\psi} D^\mu D^\nu \psi + \text{h.c.}) + \frac{c_3}{2}\langle u_\mu u^\mu \rangle \bar{\psi}\psi$$
$$- \frac{c_4}{4}\bar{\psi}\gamma^\mu \gamma^\nu [u_\mu, u_\nu]\psi + \dots \ ,$$

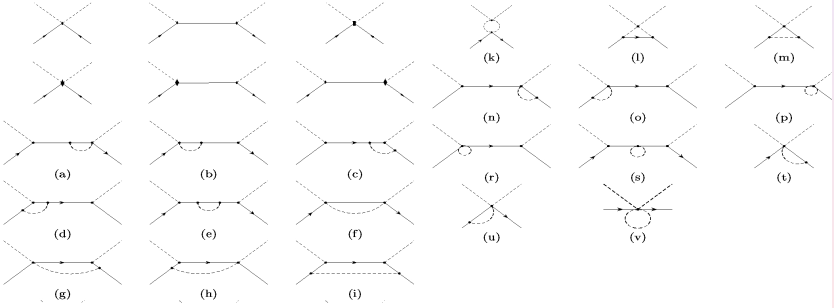
$$\mathcal{L}_{\pi N}^{(3)} = \bar{\psi}\Bigg(-\frac{d_1 + d_2}{4m}([u_\mu, [D_\nu, u^\mu] + [D^\mu, u_\nu]]D^\nu + \text{h.c.})$$
$$+ \frac{d_3}{12m^3}([u_\mu, [D_\nu, u_\lambda]](D^\mu D^\nu D^\lambda + \text{sym.}) + \text{h.c.})$$
$$+ i\frac{d_5}{2m}([\chi_-, u_\mu]D^\mu + \text{h.c.})$$
$$+ i\frac{d_{14} - d_{15}}{8m}\left(\sigma^{\mu\nu}\langle [D_\lambda, u_\mu]u_\nu - u_\mu[D_\nu, u_\lambda]\rangle D^\lambda + \text{h.c.}\right)$$
$$+ \frac{d_{16}}{2}\gamma^\mu \gamma_5 \langle \chi_+ \rangle u_\mu + \frac{i d_{18}}{2}\gamma^\mu \gamma_5 [D_\mu, \chi_-]\Bigg)\psi + \dots$$

Part III

Perturbative Calculations

From the usual power counting, we have the following contributions:

- Tree level diagrams using vertices of $\mathcal{L}_{\pi N}^{(1)}$, $\mathcal{L}_{\pi N}^{(2)}$ and $\mathcal{L}_{\pi N}^{(3)}$.
- Loop diagrams using only $\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi\pi}^{(2)}$.



Fits

We consider the phase shifts of the partial wave analyses of the Karlsruhe group **[Koch, NPA 448 (1986) 707]** (KA85) and the current one of the GWU group **[R. A. Arndt et al., PRC 74 (2006) 045205. solution SM01]** (WI08). Due to the absence of errors in these analyses there is some ambiguity in the calculation of the χ^2 so:

- We assing an error to every point as the sum in quadrature of a systematic plus a relative error.

$$\text{err}(\delta) = \sqrt{e_s^2 + e_r^2\delta^2}$$

- We take $e_r = 2\%$ as a safer estimate for isospin breaking effects (not taken into account in our study).
- And we take $e_s = 0.1$ degrees in order to stabilize fits because an $e_s = 0$ gives too much weight in the threshold region.
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Perturbative Fits

We consider two strategies to fit the KA85 and WI08 data.

- First strategy (KA85-1, WI08-1):
  - Fit phase shifts up to $\sqrt{s}_{max} = 1.13$ GeV .
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- Second strategy (KA85-2, WI08-2):
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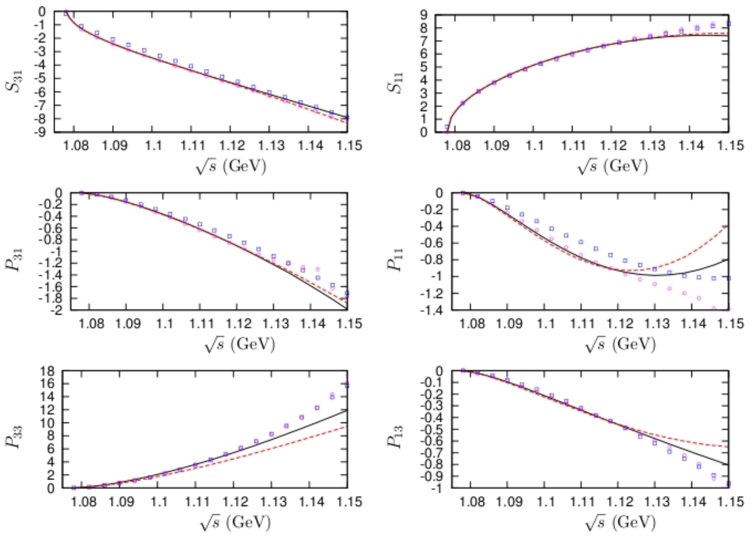
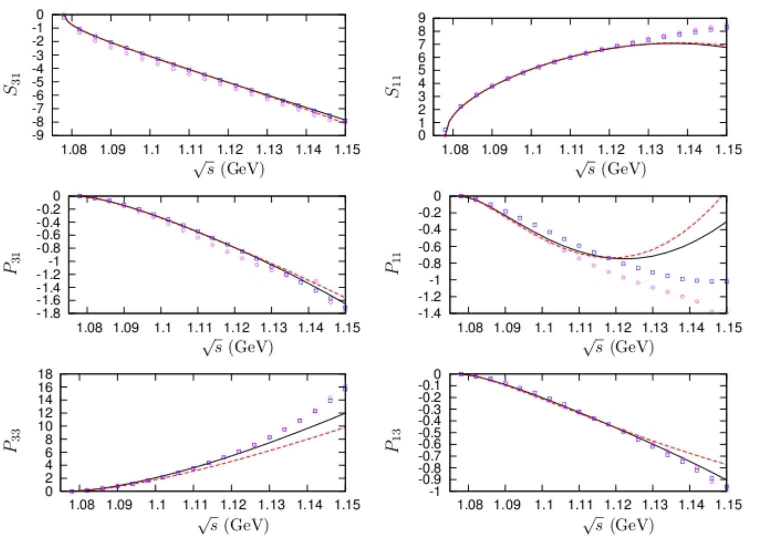
KA85 Fits



Solid line: KA85-1. Dashed line: KA85-2.

WI08 Fits



Solid line: WI08-1. Dashed line: WI08-2.

LECs Summary

Results for the LECs:

LEC	KA85-1	KA85-2	WI08-1	WI08-2	Average
c_1	-0.71 ± 0.49	-0.79 ± 0.51	-0.27 ± 0.51	-0.30 ± 0.48	-0.52 ± 0.60
c_2	4.32 ± 0.27	3.49 ± 0.25	4.28 ± 0.27	3.55 ± 0.30	3.91 ± 0.54
c_3	-6.53 ± 0.33	-5.40 ± 0.13	-6.76 ± 0.27	-5.77 ± 0.29	-6.12 ± 0.72
c_4	3.87 ± 0.15	3.32 ± 0.13	4.08 ± 0.13	3.60 ± 0.16	3.72 ± 0.37
$d_1 + d_2$	2.48 ± 0.59	0.94 ± 0.56	2.53 ± 0.60	1.16 ± 0.65	1.78 ± 1.1
d_3	-2.68 ± 1.02	-1.10 ± 1.16	-3.65 ± 1.01	-2.32 ± 1.04	-2.44 ± 1.6
d_5	2.69 ± 2.20	1.86 ± 2.28	5.38 ± 2.40	4.83 ± 2.18	3.69 ± 2.93
$d_{14} - d_{15}$	-1.71 ± 0.73	1.03 ± 0.71	-1.17 ± 1.00	1.27 ± 1.11	-0.145 ± 1.88
d_{18}	-0.26 ± 0.40	-0.07 ± 0.44	-0.86 ± 0.43	-0.72 ± 0.40	-0.48 ± 0.58

- Following a conservative procedure, the error given in the average is the sum in quarature of the largest statistical error and the one resulting from the dispersion in the central values.
- The average is compatible with those from $\mathcal{O}(p^3)$ HBChPT, except for the $d_{14} - d_{15}$ that differs by more than one standard deviation.

LECs Comparision

LEC	Average	HBCHPT $\mathcal{O}(p^3)$ [1]	HBCHPT Disp. [2]	HBCHPT $\mathcal{O}(p^3)$ [3]	RS [3]
c_1	-0.52 ± 0.60	$(-1.71, -1.07)$	-0.81 ± 0.12	-1.02 ± 0.06	
c_2	3.91 ± 0.54	$(3.0, 3.5)$	8.43 ± 56.9	3.32 ± 0.03	3.9
c_3	-6.12 ± 0.72	$(-6.3, -5.8)$	-4.70 ± 1.16	-5.57 ± 0.05	-5.3
c_4	3.72 ± 0.37	$(3.4, 3.6)$	3.40 ± 0.04		3.7
$d_1 + d_2$	1.78 ± 1.1	$(3.2, 4.1)$			
d_3	-2.44 ± 1.6	$(-4.3, -2.6)$			
d_5	3.69 ± 2.93	$(-1.1, 0.4)$			
$d_{14} - d_{15}$	-0.145 ± 1.88	$(-5.1, -4.3)$			
d_{18}	-0.48 ± 0.58	$(-1.6, -0.5)$			

[1] N. Fettes, U. G. Meißner and S. Steininger, Nucl. Phys. A 640 (1998) 199.

[2] P. Buettiker and U. G. Meißner, Nucl. Phys. A 668 (2000) 97.

[3] V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A 615 (1997) 483.

Threshold parameters summary

In order to obtain the scattering lengths and volumes we performed an effective range expansion (ERE) fit to our results in the low energy region, because numerical poblems prevent us to take directly the limit: $\lim_{|\vec{p}|\to 0}|\vec{p}|\frac{\mathrm{Re}\,T}{8\pi\sqrt{s}|\vec{p}|^{1+2\ell}}$

Partial Wave	KA85-1	KA85-2	WI08-1	WI08-2	Average
$a_{S_{31}}$	-0.100 ± 0.001	-0.103 ± 0.001	-0.081 ± 0.001	-0.082 ± 0.001	-0.092 ± 0.012
$a_{S_{11}}$	0.171 ± 0.001	0.172 ± 0.002	0.165 ± 0.002	0.167 ± 0.002	0.169 ± 0.004
a_{0+}^{+}	-0.010 ± 0.001	-0.011 ± 0.001	0.001 ± 0.001	0.001 ± 0.001	-0.005 ± 0.007
a_{0+}^{-}	0.090 ± 0.001	0.092 ± 0.001	0.082 ± 0.001	0.083 ± 0.001	0.087 ± 0.005
$a_{P_{31}}$	-0.052 ± 0.001	-0.051 ± 0.001	-0.048 ± 0.001	-0.051 ± 0.001	-0.051 ± 0.002
$a_{P_{11}}$	-0.078 ± 0.001	-0.088 ± 0.001	-0.073 ± 0.001	-0.080 ± 0.001	-0.080 ± 0.006
$a_{P_{33}}$	0.251 ± 0.002	0.214 ± 0002	0.252 ± 0.002	0.222 ± 0.002	0.232 ± 0.017
$a_{P_{13}}$	-0.034 ± 0.001	-0.035 ± 0.001	-0.032 ± 0.001	-0.035 ± 0.001	-0.034 ± 0.002

Threshold parameters comparision

Results for the threshold parameters:

Partial Wave	Average	KA85	WI08
$a_{S_{31}}$	-0.092 ± 0.012	-0.100 ± 0.004	-0.084
$a_{S_{11}}$	0.169 ± 0.004	0.175 ± 0.003	0.171
a_{0+}^{+}	-0.005 ± 0.007	-0.008	-0.0010 ± 0.0012
a_{0+}^{-}	0.087 ± 0.005	0.092	0.0883 ± 0.0005
$a_{P_{31}}$	-0.051 ± 0.002	-0.044 ± 0.002	-0.038
$a_{P_{11}}$	-0.080 ± 0.006	-0.078 ± 0.002	-0.058
$a_{P_{33}}$	0.232 ± 0.017	0.214 ± 0.002	0.194
$a_{P_{13}}$	-0.034 ± 0.002	-0.030 ± 0.002	-0.023

- None of our fits (KA85-1,KA85-2,WI08-1,WI08-2) is compatible with the value of $a_{P_{11}}$ given by WI08

Goldberger-Trieman relation

The value of d_{18} is important because is directly related to the violation of the Goldberger-Trieman (GT) relation. One has, up to $\mathcal{O}(M_\pi^3)$:

$$g_{\pi N} = \frac{g_A m}{F_\pi}\left(1 - \frac{2M_\pi^2 d_{18}}{g_A}\right)$$

We quantify the deviaton from the GT relation by:

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$$\Delta_{GT} = 0.015 \pm 0.018$$

Which is compatible with the values around $2-3\%$ obtained from πN and NN partial wave analyses **[Arndt, Workman and Pavan, PRC 49 (1994) 2729]**, **[Schröder et al]**,**[Swart, Rentmeester and Timmermans, πN Newsletter 13 (1997)96]**.
This value of Δ_{GT} gives:

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But when we implement the loop contribution, we obtain a huge GT relation violation:

- For the fit KA85-1 one has a 22% of violation for $\mu = 1$ GeV (scale) while for $\mu = 0.5$ GeV a 15% stems.

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Part IV

Unitarized Calculations

Unitarized Calculations

In order to implement unitarity to the πN amplitude and take care of the analyticity properties associated with the right-hand cut we write our unitarized amplitude $T_{IJ\ell}$ by means of an interaction kernel $\mathcal{T}_{IJ\ell}$ and the unitary pion-nucleon loop function $g(s)$:

$$T_{IJ\ell} = \frac{1}{\mathcal{T}_{IJ\ell}^{-1} + g(s)}$$

- $T_{IJ\ell}$ satisfies unitarity exactly.
- The interaction kernel is determined order by order by matching with the perturbative ChPT result **[J. A. Oller and U. G. Meißner, PLB 500:263-272 (2001)]**.
- a_1 is fixed by requiring $g(m^2) = 0$ (in order to have the P_{11} nucleon pole in its right position).

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We introduce the contribution of the $\Delta(1232)$ in P_{33} through a CDD **[Castillejo, Dalitz and Dyson, PR 101 (1956) 453]**, **[Oller and Oset, PRD 60, 074023 (1999)]**:

- The CDD pole conserves the discontinuities of the partial wave amplitude across the cuts.
- The CDD pole corresponds to a zero of the partial wave amplitude along the real axis and hence to a pole in the inverse of the amplitude.

$$T_{\frac{3}{2}\frac{3}{2}1} = \left(\mathcal{T}^{-1}_{\frac{3}{2}\frac{3}{2}1} + \frac{\gamma}{s - s_P} + g(s) \right)^{-1}$$

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IR regularization introduces unphysical cuts due to the infinite order resummation of the sub-leading $1/m$ kinetic energy when $u = 0$, that correspond to $s = 2(m^2 + M_\pi^2) \gtrsim 1.34^2$ GeV^2. Consequences:

- Strong violation of unitarity.
- Strong rising of the phase-shifts from energies $\sqrt{s} \gtrsim 1.26$ GeV.

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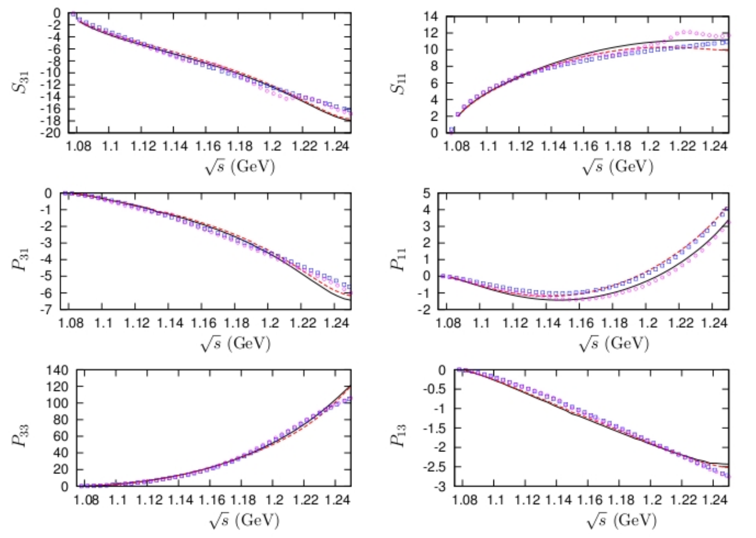

Solid line: Fit to KA85 data. Dashed line: Fit to WI08 data.

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- We obtain a good agreement with data in the whole energy range form threshold up to 1.25 GeV.
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LEC	Fit KA85	Fit WI08	Average (Perturbative)	Partial Wave	Fit KA85	Fit WI08	Average (Perturbative)
c_1	-0.48 ± 0.51	-0.52 ± 0.60	-0.53 ± 0.48	$a_{S_{31}}$	-0.115	-0.104	-0.092 ± 0.012
c_2	4.62 ± 0.27	4.73 ± 0.30	3.91 ± 0.54	$a_{S_{11}}$	0.152	0.150	0.169 ± 0.004
c_3	-6.16 ± 0.27	-6.41 ± 0.29	-6.12 ± 0.72	a^+_{0+}	-0.026	-0.020	-0.005 ± 0.007
c_4	3.68 ± 0.13	3.81 ± 0.16	3.72 ± 0.37	a^-_{0+}	0.089	0.085	0.087 ± 0.005
$d_1 + d_2$	2.55 ± 0.60	2.70 ± 0.65	1.78 ± 1.1	$a_{P_{31}}$	-0.050	-0.048	-0.051 ± 0.002
d_3	-1.61 ± 1.01	-1.73 ± 1.04	-2.44 ± 1.6	$a_{P_{11}}$	-0.080	-0.075	-0.080 ± 0.006
d_5	0.93 ± 2.40	1.13 ± 2.18	3.69 ± 2.93	$a_{P_{33}}$	0.245	0.250	0.232 ± 0.017
$d_{14} - d_{15}$	-0.46 ± 1.00	-0.61 ± 1.11	-0.145 ± 1.88	$a_{P_{13}}$	-0.41	-0.039	-0.034 ± 0.002
d_{18}	0.01 ± 0.21	-0.03 ± 0.20	-0.48 ± 0.58				

Unitarized Calculations

- The values of these LECs do not constitute an alternative determination to the perturbative results.
- These values only should be employed within UChPT studies.
- LECs and threshold parameters compatible with the average values given in the perturbative calculation.
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Part V

Summary and Conclusions

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- Perturbative calculations:
  - We used two sets of data (form Karlsruhe and GWU groups) to fit our theorical result.
  - An accurate reproduction of the phase-shifts was obtained up to 1.14 GeV, similar in quality to that obtained previously with $\mathcal{O}(p^3)$ HBChPT $\Rightarrow$ **Improvement** compared with previous works.
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EOMS

But we still have one possible solution for the limitations of IR: The Extended-On-Mass-Shell scheme (EOMS),
[Fuchs, Gegelia, Japaridze and Scherer, PRD 68, 056005 (2003)].

- This scheme removes explicity the power counting breaking terms appearing in the loop integrals in dimensional regularization.
- These PCBT terms are absorbed in the LECs (IR result).
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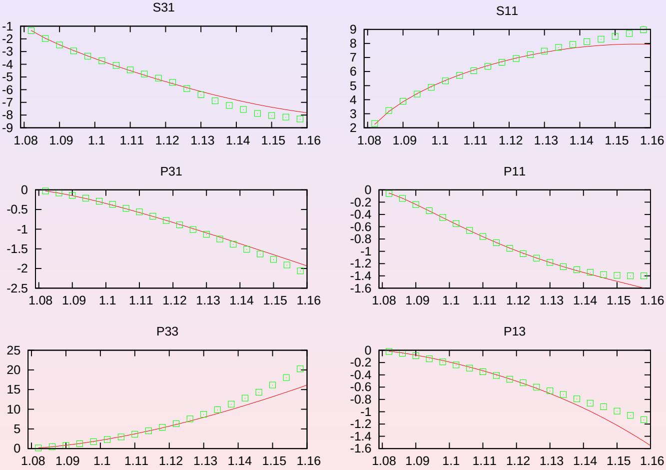
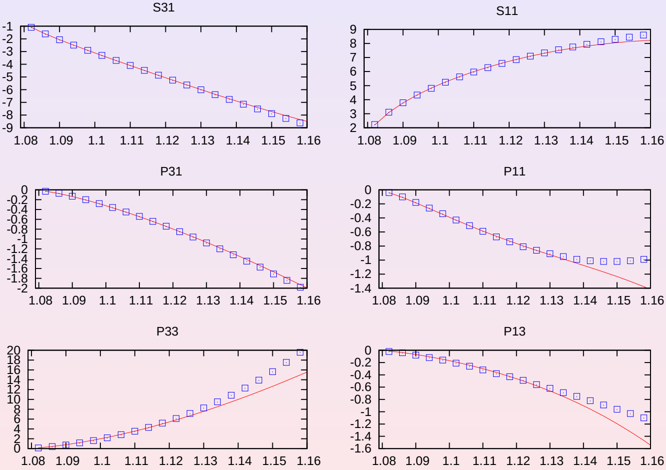
EOMS-KA85



EOMS-WI08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1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16
S11
9 8 7 6 5 4 3 2
1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16
P31
0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4 -1.6 -1.8 -2
1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16
P11
0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4
1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16
P33
20 18 16 14 12 10 8 6 4 2 0
1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16
P13
0 -0.2 -0.4 -0.6 -0.8 -1 -1.2 -1.4 -1.6
1.08 1.09 1.1 1.11 1.12 1.13 1.14 1.15 1.16

FIN

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